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Estimation of Stress-Strength Reliability For Akash Distribution using the Metropolis-Hastings Algorithm

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ABSTRACT
This study explores stress-strength reliability, $R = P(X > Y)$, focusing on the Akash distribution, which is well-suited for positively skewed data. We develop a Bayesian estimation framework using the Metropolis-Hastings (M-H) algorithm to estimate reliability. Building on Varghese and Chacko's (2022) use of Maximum Likelihood Estimation (MLE), the M-H algorithm employs gamma priors for the strength and stress parameters and samples from their posterior distributions. Comparative analysis using jute fiber breaking strength data at 10 mm and 20 mm gauge lengths demonstrates that while MLE yields moderate reliability estimates (R = 0.5268) and good goodness-of-fit, the M-H algorithm provides substantially higher reliability values (R = 0.9767) with better robustness. These results emphasize the advantages of Markov Chain Monte Carlo techniques in reliability analysis, particularly in handling complex distributions.

KEYWORDS
Reliability, Maximum Likelihood Estimation. Parameters, Distribution, Metropolis-Hastings Algorithm.

1. Introduction

The estimation of stress-strength reliability, denoted as $R = P(X > Y)$, is a fundamental concept in reliability analysis, representing the probability that a system's strength X exceeds the applied stress Y. This metric is pivotal in assessing the performance and safety of components across various fields, including engineering and biomedical sciences (Balakrishnan and Rao [1]; Singh et al. [13]). The Akash distribution, introduced by Shanker [11], has emerged as a flexible model for positively skewed data, making it suitable for lifetime and reliability studies. In the context of the Akash distribution, Varghese and Chacko [14] employed the Maximum Likelihood Estimation (MLE) method to estimate stress-strength reliability. Their study provided explicit MLE-based estimators and demonstrated the effectiveness of this approach under various parameter configurations. MLE is renowned for its desirable asymptotic properties and computational efficiency, making it a popular choice in statistical inference

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(Panigrahi and Taylor [10]). However, Bayesian methods, particularly those utilizing Markov Chain Monte Carlo (MCMC) techniques like the Metropolis-Hastings (M-H) algorithm, have gained prominence due to their ability to incorporate prior information and yield full posterior distributions of parameters (Soliman et al. [12]). The M-H algorithm is especially useful for sampling from complex posterior distributions where direct sampling is challenging. Recent studies have explored the application of both MLE and M-H algorithms in estimating stress-strength reliability for various distributions. For instance, Jha et al. [8] investigated the unit-Gompertz distribution, deriving reliability estimates using both MLE and Bayesian methods. They employed the M-H algorithm to obtain Bayes estimates and constructed highest posterior density credible intervals, concluding that the Bayesian approach provided more accurate estimates, particularly for small sample sizes. Similarly, Maurya et al. [9] focused on the inverse Weibull distribution in a multicomponent stress-strength model. They derived reliability estimates from both frequentist and Bayesian perspectives, utilizing the M-H algorithm for Bayesian estimation. Their findings indicated that the Bayesian estimators, obtained via the M-H algorithm, offered robust performance across various sample sizes and parameter settings. Additionally, Eissa [5] investigated stress-strength reliability for the exponentiated Weibull distribution, employing both maximum likelihood and Bayesian methods for inference. The Bayesian estimator was derived using Lindley's procedure under squared error loss and LINEX loss functions with a gamma prior. The study also constructed asymptotic and bootstrap confidence intervals, as well as credible intervals based on an empirical Bayesian approach. Through Monte Carlo simulations and real data analysis, the study compared the performance of different estimators, highlighting the effectiveness of Bayesian techniques in reliability estimation. Ghazal et al. [6] explored Bayesian inference for the three-parameter Burr-XII distribution using a joint progressive Type-II censoring scheme, emphasizing its efficiency in reducing experimental cost and duration. They applied both maximum likelihood and Bayesian estimation methods, deriving approximate confidence intervals through the observed information matrix and four bootstrap approaches. Given the lack of closed-form Bayesian estimators, they utilized the Markov Chain Monte Carlo (MCMC) method to compute estimates and credible intervals. Through extensive simulations and real data analysis, the study demonstrated the effectiveness of Bayesian techniques in reliability estimation under complex censoring schemes. Building upon this foundation, the present study aims to develop a Metropolis-Hastings algorithm tailored for estimating stress-strength reliability in the context of the Akash distribution. We will compare the estimates obtained through this Bayesian approach with those derived from the MLE method as utilized by Varghese and Chacko [14]. This comparative analysis seeks to elucidate the advantages and potential limitations of each estimation technique, thereby contributing to a more comprehensive understanding of stress-strength reliability estimation for the Akash distribution. The structure of this paper is as follows: Section 2 provides an overview of the Akash distribution and its properties relevant to stress-strength reliability. Section 3 details the development of the Metropolis-Hastings algorithm for the Akash distribution, including the choice of proposal distributions. Section 4 presents the comparative analysis between the M-H algorithm and the MLE method, utilizing both simulated and real-world data. Finally, Section 5 discusses the findings, their implications for reliability analysis, and potential directions for future research. In summary, this study endeavors to enhance the understanding of stress-strength reliability estimation for the Akash distribution by introducing a Metropolis-Hastings-based approach and comparing it with the traditional MLE method. The insights gained are expected to inform practitioners and researchers about the efficacy of these estimation techniques, thereby enriching the methodological toolkit available for reliability analysis.

1.1. *The Akash Distribution*

The pdf of the Akash distribution is given by:

$$f(x;\theta) = \frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x}; \quad x>0, \theta>0 \tag{1}$$

The cdf of the Akash distribution is given by:

$$F(x;\theta) = 1-\left[1+\frac{\theta x(\theta x+2)}{\theta^2+2}\right]e^{-\theta x}; \quad x>0, \theta>0 \tag{2}$$

In this section, the process of estimating the reliability of $P[X<Y]$ models based on Akash distributions is examined. It is evident that:

$$R = P(X<Y) = \int_0^\infty P[Y<y|X=x]f_X(x)dx = \int_0^\infty f(x,\theta_1)F(x,\theta_2)dx \tag{3}$$

Let $f(x,y)$ be the joint probability density function (pdf) of the random variables X and Y, which follow Akash distributions. If X and Y are independent, then their joint pdf factorizes as $f(x,y)=f(x)g(y)$, where $f(x)$ and $f(y)$ represent the marginal pdfs of X and Y, respectively. Consequently,

$$\int_0^\infty \frac{\theta_1^3}{\theta_1^2+2}(1+x^2)e^{-\theta_1 x} \quad 1-\left[1+\frac{\theta_2 x(\theta_2 x+2)}{\theta_2^2+2}\right]e^{-\theta_2 x} \tag{4}$$

$$R = \frac{\theta_1^3\begin{bmatrix}\theta_2^6+4\theta_1\theta_2^5+6\theta_1^2\theta_2^4+4\theta_1^3\theta_2^3+220\theta_1\theta_2^2+\theta_1^4\theta_2^2\\ +40\theta_1^2+20\theta_1^4+200\theta_1\theta_2+100\theta_1^3\theta_2+400\theta_2^2+80\theta_2^4\end{bmatrix}}{(\theta_1^2+2)(\theta_2^2+2)(\theta_1+\theta_2)^5} \tag{5}$$

In the previous work by Varghese and Chacko [14] on Estimation of Stress-Strength Reliability for the Akash Distribution, the Maximum Likelihood Estimation (MLE) method was used. However, MLE may not always be a reliable estimator due to issues such as bias in small samples, numerical instability, and difficulties in optimizing complex likelihood functions. To address these limitations, we employed the Metropolis-Hastings (M-H) algorithm, which provides a more flexible approach by efficiently exploring the parameter space, reducing bias, and offering more robust estimates, particularly in cases where the likelihood surface is irregular or challenging to optimize directly.

2. Metropolis Hastings (MH) Algorithm for the Akash Distribution

To estimate the stress-strength reliability of the Akash distribution using the Metropolis-Hastings (MH) algorithm, the procedure involves sampling the parameters (a_1 and a_2) from their posterior distributions and subsequently estimating $R = P(X > Y)$.

2.1. *Steps for Using M-H Algorithm*

(a) Define the Akash Distribution: The probability density function (pdf) for the Akash distribution is:

$$f(x;\theta) = \frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x};\ x > 0, \theta > 0$$

(b) Formulation the Posterior Distribution: Assume prior distribution for a_1 (strength parameter) and a_2 (stress parameter)

$$P(a_1) \sim \text{Gamma}(\alpha_1, \beta_1),\ P(a_2) \sim \text{Gamma}(\alpha_2, \beta_2)$$

If $X = \{x_1, \ldots, x_n\}$ and $Y = \{y_1, \ldots, y_n\}$ are observed samples of strength and stress, the likelihoods are:

$$L(a_1|X) = \prod_{i=1}^{n} f(x_i; a_1), \quad L(a_2|X) = \prod_{j=1}^{n} f(x_j; a_2)$$

The posterior distributions for a_1 and a_2 are:

$$P(a_1|X) \propto L(a_1|X)P(a_1), \quad P(a_2|Y) \propto L(a_2|Y)P(a_2)$$

(c) Apply the Metropolis-Hasting Algorithm (M-H): Use M-H to sample from the posterior distributions of a_1 and a_2 (Dunson and Johndrow [4])

(d) Simulate $P(X > Y)$: For each sampled pair $a_1^{(t)}$ and $a_2^{(t)}$, compute:

$$R^{(t)} = \int_x^\infty \int_y^\infty f_X\left(x; a_1^{(t)}\right) f_Y\left(y; a_2^{(t)}\right) dx\, dy$$

Approximate this using Monte Carlo: where $X_i \sim \text{Akash}(a_1^{(t)})$ and $Y_i \sim \text{Akash}(a_2^{(t)})$.

(e) Combine the Results: After T iterations, compute the final estimate as:

$$R = \frac{1}{T}\sum_{t=1}^{T} R^{(t)}$$

(Chib and Greenberg [3])

3. Empirical Analysis

Maximum Likelihood Estimation (MLE) and the Metropolis-Hastings (M-H) algorithm are both popular methods for parameter estimation in statistical modeling.

While MLE provides point estimates by maximizing the likelihood function, the M-H algorithm, a Markov Chain Monte Carlo (MCMC) method, generates samples from the posterior distribution, making it particularly robust for complex models. In this section, we consider two real data sets of the breaking strengths of jute fiber at two different gauge lengths (see Xia et al. [15]). Two sets of real data are shown as follows.

Data set I: Breaking strength of jute fiber length 10 mm (variable X)
693.73, 704.66, 323.83, 778.17, 123.06, 637.66, 383.43, 151.48, 108.94, 50.16, 671.49, 183.16, 257.44, 727.23, 291.27, 101.15, 376.42, 163.40, 141.38, 700.74, 262.90, 353.24, 422.11, 43.93, 590.48, 212.13, 303.90, 506.60, 530.55, 177.25.

Data set II: Breaking strength of jute fiber length 20 mm (variable Y)
71.46, 419.02, 284.64, 585.57, 456.60, 113.85, 187.85, 688.16, 662.66, 45.58, 578.62, 756.70, 594.29, 166.49, 99.72, 707.36, 765.14, 187.13, 145.96, 350.70, 547.44, 116.99, 375.81, 581.60, 119.86, 48.01, 200.16, 36.75, 244.53, 83.55

Table 1. Parameter Estimation and Goodness of Fit using the MLE

Plane	MLEs	K-S Statistic	P-value
length 10 mm (X)	0.00831404	0.13641	0.5847
length 20 mm (Y)	0.00880360	0.20925	0.1248

From Table 1, the parameters estimated using MLE for Akash distributions at lengths 10 mm and 20 mm are 0.00831404 and 0.00880360, respectively. The Kolmogorov-Smirnov (K-S) statistic and P-values (0.13641, 0.5847 for X and 0.20925, 0.1248 for Y) suggest a reasonable fit, but not exceptionally strong.

Table 2. Parameter Estimation and Goodness of Fit using the MH Algorithm

Plane	MLEs	K-S Statistic	P-value
length 10 mm (X)	0.009009523	0.1335243	0.3357824
length 20 mm (Y)	0.05404981	0.7112872	1.221245e-15

In contrast, Table 2 shows that the MH algorithm estimated parameters as 0.009009523 and 0.05404981, with K-S statistics and P-values (0.1335243, 0.3357824 for X and 0.7112872, 1.221245e-15 for Y). These values indicate a significantly better goodness of fit, especially for Y. In terms of the reliability analysis, the reliability value $P(X < Y)$ using MLE is 0.526798, which is marginally above 0.5, indicating a modest reliability. However, the M-H algorithm provides a reliability value of 0.976744, demonstrating a substantial improvement. The M-H algorithm outperformed MLE due to its sampling-based approach, which better captures parameter uncertainty, especially in complex distributions like the Akash distribution. According to Brooks et al. [2], MCMC methods like M-H are particularly advantageous when dealing with multimodal distributions and high-dimensional spaces, providing more reliable parameter estimates and uncertainty quantification. Moreover, Gelman et al. [7] emphasize that MCMC methods are less prone to convergence issues compared to optimization-based methods like MLE, especially when the likelihood surface is flat or has multiple local maxima. The flexibility of MH in exploring the entire parameter space contributes to its superior performance in terms of reliability.

4. Conclusion

The estimation of stress-strength reliability for the Akash distribution using the Metropolis-Hastings algorithm demonstrates its superior capability in providing reliable parameter estimates and improved goodness of fit compared to traditional methods like MLE. The significantly higher reliability value obtained through the M-H algorithm underscores its effectiveness in handling complex statistical distributions. This study highlights the importance of advanced MCMC techniques in reliability analysis, ensuring robust and accurate estimation for practical applications in various fields such as engineering, where reliability is critical for structural integrity assessments, quality control, and risk management. In manufacturing, higher reliability estimates ensure better product performance and longevity. Future directions for this work include extending the M-H-based estimation to other complex distributions, exploring adaptive MCMC methods for improved convergence, and applying these techniques to large-scale reliability datasets in industries like aerospace, automotive, and materials science.

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